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Cooperative and non-cooperative fiscal stabilization policies in the EMU

Jacob C. Engwerda^{1,5}, Bas van Aarle^{2,3}, Joseph E.J. Plasmans^{1,4}

Abstract

In this paper we analyze the interaction of fiscal stabilization policies in the Economic and Monetary Union (EMU). The “Excessive Deficits” Procedure of the Maastricht Treaty and its elaborations in the recent “Stability and Growth Pact” introduce a set of fiscal stringency requirements on national fiscal policies. Situations might arise where the need for fiscal flexibility and the fiscal stringency requirements will create a conflict and suboptimal macroeconomic policies are implemented. We analyze macroeconomic adjustment under non-cooperative and cooperative fiscal policy design in the EMU using a dynamic games approach. In particular we consider how fiscal stringency requirements like the Stability and Growth Pact affect fiscal policy design under EMU and study the consequences of the introduction of a fiscal transfer mechanism between countries.

JEL classification: C7; E6; F4

Keywords: Fiscal Policy Design, EMU, Dynamic Games

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1 Introduction

In the Economic and Monetary Union (EMU), the participating countries loose monetary and exchange rate policies as macroeconomic policy instruments. The EMU implies a considerable change in the design of macroeconomic policies, both at the national and the supranational level. Monetary policy design is transferred to the European Central Bank (ECB) that implements the common monetary policy and circulates the common currency, the Euro. The monetary policies of the ECB are mainly directed at price stability in the EU and maintaining a stable external value of the Euro. In the short and medium term, the burden of fiscal stabilization primarily rests on the national fiscal authorities given also the small size of the federal EU budget. This situation is likely to increase the need for fiscal policy activism when countries face a recession.

A first important issue regarding fiscal policy design in the EMU concerns the need for fiscal policy coordination. EMU affects both the interaction of fiscal policies and their transmission in the EU economies. Given the high degree of economic interdependence in the EU, important externalities from national fiscal policies exist. Coordination of national fiscal policies enables to internalize these externalities and by that to improve macroeconomic performance compared to non-cooperative fiscal policy design in the EMU.

A second important issue concerns the imposed fiscal stringency requirements by the “Excessive Deficits Procedure” of the Maastricht Treaty and its detailed elaboration convened in the “Stability and Growth Pact” (Stability Pact in short) that was signed at the June 1997 Amsterdam summit of the Council of EU ministers. It imposes a set of restrictions on fiscal flexibility under EMU. The Stability Pact has a double role: (i) a preventive role of early warning against excessive budget deficits (budget surveillance), and (ii) a penalizing role for sustained budget shortages. The medium term goal is approximate budget equilibrium or budget surplus. It was motivated by the fear that undisciplined fiscal behavior is likely to put at risk the low inflation commitment of the ECB since it will be difficult to rule out a monetary bail-out by the ECB under all circumstances. Undisciplined fiscal behavior may also result in fiscal bail-outs through fiscal transfers in the EU. Finally, excessive deficits could induce upward pressure on interest rates and an appreciation of the Euro. In both cases, pressure on the ECB could arise to ease its monetary policy. In all cases, the burdens associated with individual fiscal indiscipline will partly be transmitted to the other EU countries.

Situations, however, may arise where the need for greater fiscal flexibility and the greater fiscal stringency will create a conflict and suboptimal macroeconomic policies will be pursued. This paper analyzes the interaction of fiscal policies in the EMU and shows how fiscal stringency criteria like the Stability Pact affect this interaction and macroeconomic adjustment. To do so, we analyze outcomes in two different game-theoretic settings: (i) non-cooperative fiscal policy design and (ii) cooperative fiscal policy design. We also analyze how policies and adjustment are affected in both regimes by externally imposed fiscal stringency measures and consider the case for a European fiscal transfer system.

To model the design of fiscal stabilization policies under EMU, we introduce a dynamic two-country model of the EMU that features short term nominal rigidities thus creating scope for active stabilization policies. Our analysis builds on earlier work by Turnovsky, Başar and d'Orey (1988) and Neck and Dockner (1995) who analyze the interaction of monetary authorities in a similar dynamic two country model. In both papers, the monetary policies of the two countries affect short-term output in both the domestic and foreign economies. The interdependencies of the economies, hence, creates a dynamic conflict between the two monetary authorities. Output, inflation and exchange rate adjustment and their implications for social welfare are calculated for a number of different modes of strategic interaction. We extend these two-country models into a setting of a monetary union and consider the effects of fiscal policy in such a setting of a monetary union and analyze the effects of fiscal stringency conditions and fiscal transfers on the outcomes. In order to be able to derive analytical properties of the model we concentrate on open-loop strategies in the dynamic game between the fiscal players.

The paper is organized as follows: section 2 develops the analytical framework, section 3 analyzes non-cooperative and cooperative fiscal policies under EMU, section 4 presents numerical simulations of the model to illustrate its main characteristics, and the final section concludes.

2 A Dynamic Stabilization Game in the EMU

Consider a situation where a two-country EMU has been fully implemented, implying that national currencies have been replaced by a common currency, national central banks by the ECB and that the internal exchange rate has disappeared as an adjustment instrument. Furthermore assume that capital

markets are fully integrated and that there are no country-risk premia implying that any interest differential is arbitrated away instantaneously. On the other hand assume that there is no labor mobility between both EMU parts and that goods and labor market adjust sluggishly. That is, the economies display Keynesian features in the short-run.

We model this economic structure of a two-country EMU by the following equations,

Table 1: A Stylized Two-Country EMU Model

$$y(t) = \delta s(t) - \gamma i_r(t) + \rho y^*(t) + \eta f(t) \quad (1)$$

$$y^*(t) = -\delta s(t) - \gamma i_{r^*}(t) + \rho y(t) + \eta f^*(t) \quad (2)$$

$$s(t) = p^*(t) - p(t) \quad (3)$$

$$i_r(t) = i^E(t) - \dot{p}(t) \quad (4)$$

$$i_{r^*}(t) = i^E(t) - \dot{p}^*(t) \quad (5)$$

$$m(t) - p(t) = \kappa y(t) - \zeta i^E(t) \quad (6)$$

$$m^*(t) - p^*(t) = \kappa y^*(t) - \zeta i^E(t) \quad (7)$$

$$\dot{p}(t) = \xi y(t) \quad (8)$$

$$\dot{p}^*(t) = \xi y^*(t) \quad (9)$$

in which, y , denotes real output, p , the output price level, i^E , the common nominal interest rate and, i_r , the real interest rate. s measures competitiveness of country 1 vis-à-vis country 2 and is defined as the output price differential. f , equals the real fiscal deficit that the fiscal authority sets. m denotes the amount of nominal money balances that the public demands.

Except for the nominal interest rate and the rate of inflation, \dot{p} , variables are in logarithms and expressed as deviations from their long-run non-inflationary equilibrium (growth path). The model, therefore, characterizes the business cycles in this two-country EMU. Variables of country 2 are indicated with an asterisk. For simplicity, we assume that both countries are symmetric in their structural model parameters and we ignore the interaction of this two-country EMU with the rest of the world.

(1) is the aggregate demand function having intra-EU competitiveness, the real interest rate, foreign output and the fiscal deficit as arguments. (3) defines the competitiveness of the EMU countries relative to each other. The

definition of the real interest rate is given in (4). The demand for real balances of the common currency is given in (6) as a function of output and the common nominal interest rate. We assume that its interest targeting policy enables the ECB to have perfect control over the nominal common interest rate, $i^E(t)$. Given our focus on fiscal policies, we assume in the remainder that the ECB pursues a fixed interest rate policy implying $i^E(t) = \bar{i}^E$. Note, however, that also alternative interest rate rules like a Taylor rule could be introduced. (8), finally, gives the short run relation between inflation and output, along the Phillips curve. Because of the nominal rigidities, implied by the Phillips curve, output and prices can diverge from their equilibrium values in the short run. In the long run, on the other hand, both economies adjust to a long run equilibrium where output and prices are at their equilibrium values (which have been normalized to zero in this analysis). Both economies are connected by a number of channels through which price and output fluctuations in one part transmit themselves to the other part of the EMU. Output fluctuations in both economies transmit themselves partly to the other EMU country through the import channel. Therefore, the relative openness of both economies, as measured by ρ , implies an important interdependence of both economies. Price fluctuations in the domestic or foreign economy affect intra-EU competitiveness, $s(t)$, and therefore output in both economies. Combining (1)-(9), enables to write output in both countries as a function of competitiveness, the policy instrument of the ECB, $i^E(t)$, and the fiscal deficit set by the two fiscal authorities, $f(t)$ and $f^*(t)$,

$$y(t) = bs(t) - ci^E(t) + af(t) + \frac{\rho}{k}af^*(t) \quad (10)$$

$$y^*(t) = -bs(t) - ci^E(t) + \frac{\rho}{k}af(t) + af^*(t) \quad (11)$$

with $a := \frac{\eta k}{k^2 - \rho^2}$, $b := \frac{\delta}{k + \rho}$, $c := \frac{\gamma}{k - \rho}$ and $k := 1 - \gamma\xi$. Substituting (10) and (11) into (8) and (9) yields two first-order linear differential equations in the output price levels. Subtracting them from each other gives the dynamics of intra-EU competitiveness,

$$\dot{s}(t) = \phi_1 f^*(t) - \phi_1 f(t) + \phi_2 s(t) \quad (12)$$

with $\phi_1 := \frac{\xi\eta}{k + \rho}$ and $\phi_2 := -\frac{2\delta\xi}{k + \rho}$.

Having modeled the economies of both EMU countries and derived the adjustment dynamics of output and prices over time, we still need to determine

the fiscal policies and their dynamic adjustment over time as a consequence of the different modes of interaction of these macroeconomic policymakers. In order to do so, we need to specify the objective functions of the players. The objectives are optimized subject to the dynamics of s in (12). We assume that the players have quadratic objective functions. The dynamic strategic interaction of the policymakers in that case reduces to a linear-quadratic (LQ) differential game.

In particular, both fiscal authorities seek to minimize the following intertemporal loss functions that are assumed to be quadratic in the rate of inflation, output and fiscal deficits,

$$J^F = \frac{1}{2} \int_0^\infty \{\alpha p^2(t) + \beta y^2(t) + \chi f^2(t)\} e^{-\theta t} dt \quad (13)$$

$$J^{F*} = \frac{1}{2} \int_0^\infty \{\alpha p^{*2}(t) + \beta y^{*2}(t) + \chi f^{*2}(t)\} e^{-\theta t} dt. \quad (14)$$

Future losses are discounted at a rate θ . The costs of price and output fluctuations are standard in most analysis of macroeconomic policy design. The assumption that the fiscal authorities also value budget balance reflects the notion that high deficits, while beneficial to stimulate domestic output, are not costless: they to some extent crowd out private investment and lead to debt accumulation. Deficits in the loss function also features the possibility that excessive deficits in the EMU will be subject to sanctions, as proposed in the Stability Pact. Therefore, countries prefer low fiscal deficits, *ceteris paribus*, to high fiscal deficits. In case where $\chi \rightarrow \infty$, (cyclical) budget balance becomes the sole objective of the fiscal authority and fiscal activism is reduced accordingly. On the other hand, $\chi \rightarrow 0$, implies that fiscal stringency is minimal and that the fiscal authorities have maximal fiscal flexibility under EMU.

We consider the dynamic adjustment process caused by an initial disequilibrium in intra-EU competitiveness, implying that $s(0) \neq 0$. This initial disequilibrium causes asymmetric adjustment of output, prices and optimal policies in the adjustment towards equilibrium. We analyze how fiscal policies adjust over time as a result of the dynamic interaction between the macroeconomic policymakers in the EMU. In this dynamic interaction we focus on the different adjustment patterns that arise under non-cooperative and cooperative fiscal policy design in the EMU and how these patterns are affected by different degrees of fiscal stringency and the introduction of a federal fiscal transfer system. As mentioned in the introduction, we focus

on the open-loop strategies of this game in order to be able to derive some analytical results in the next section.

3 Non-cooperative versus Cooperative Fiscal Policies in the EMU

3.1 The non-cooperative case

We first analyze the design of fiscal policy in the EMU if the fiscal authorities implement non-cooperative fiscal policy strategies. In a Nash equilibrium setting the players implement their optimal strategies simultaneously. The optimization problems of the fiscal players in that case can be written as,

$$\begin{aligned} \min_{u_i(t)} J_i &= \frac{1}{2} \int_0^\infty \{ [x(t)^T \ u_1^T(t) \ u_2^T(t)]^T F_i \begin{bmatrix} x(t) \\ u_1(t) \\ u_2(t) \end{bmatrix} \} dt \\ \text{s.t. } \dot{x}(t) &= Ax(t) + B_1 u_1(t) + B_2 u_2(t), \\ x(0) &= x_0, \ i = 1, 2 \end{aligned} \tag{15}$$

in which

$$x(t) := e^{-\frac{1}{2}\theta t} \begin{bmatrix} s(t) \\ i^E(t) \end{bmatrix}, \ u_1(t) := e^{-\frac{1}{2}\theta t} \begin{bmatrix} f_1(t) \\ 0 \end{bmatrix}, \ u_2(t) := e^{-\frac{1}{2}\theta t} \begin{bmatrix} f_2(t) \\ 0 \end{bmatrix}.$$

The system parameters are

$$A := \begin{bmatrix} \phi_2 - \frac{1}{2}\theta & 0 \\ 0 & -\frac{1}{2}\theta \end{bmatrix}, \ B_1 := \begin{bmatrix} -\phi_1 \\ 0 \end{bmatrix}, \ \text{and} \ B_2 := \begin{bmatrix} \phi_1 \\ 0 \end{bmatrix}$$

and F_i is a positive semi-definite matrix that can be factorized as,

$$F_i =: \begin{bmatrix} Q_i & P_i & L_i \\ P_i^T & R_{1i} & N_i \\ L_i^T & N_i^T & R_{2i} \end{bmatrix}$$

in which Q_i, P_i, L_i, N_i and R_{ii} , ($i = 1, 2$), represent submatrices that are given in Appendix I.

Using the symbolic computational program Mathematica, it is shown in Appendix I that (depending on the sign of the λ_i 's, see (22)) either the game has no solution or the closed-loop system dynamics satisfy the relationship

$$\dot{x}(t) = \begin{bmatrix} -\lambda_i & 0 \\ 0 & -\frac{1}{2}\theta \end{bmatrix} x(t) \quad (16)$$

where i equals 1 or 2. λ is the adjustment speed of the output price differential, $s(t)$, towards its long-run equilibrium value zero. In other words if the game has a solution then, in principle, two different adjustment schemes of the closed-loop system (16) towards its long-term equilibrium can occur. Assuming that the parameter k is positive and denoting $\frac{\rho}{k}$ by r , $\frac{\mu(\theta k + 2\delta\xi)\eta^2\theta}{(\rho-k)(4\delta\xi + \theta(k+\rho))^2}$ by χ_1 and $\frac{\rho-k}{k}\mu\left(\frac{\eta k}{k^2-\rho^2}\right)^2$ by χ_2 , the next table illustrates the possibilities

Table 2

# equilibria	parameter values
1	$r \leq 1$
0	$\chi_1 < \chi < \chi_2, r > 1$
1	$\chi \leq \min(\chi_1, \chi_2), r > 1$
1	$\chi \geq \max(\chi_1, \chi_2), r > 1$
2	$\chi_2 < \chi < \chi_1, r > 1$

Given our model we expect, normally, that $k > \rho$ will hold. In that case, the domestic fiscal instrument has a stronger impact on domestic output than the foreign fiscal instrument (see 10). That is, $r \leq 1$ and therefore the closed-loop adjustment scheme will be uniquely determined. In the following Figure 1 we illustrated the situations that can occur in case $k < \rho$



Figure 1

In particular note that if ρ is much larger than k the situation occurs that the game permits two different types of adjustment schemes for the closed-loop system if χ is chosen "appropriately". Which adjustment scheme actually will occur under these circumstances depends on additional requirements which are imposed on the outcome of the game. A natural choice seems to select that outcome of the game that increases the adjustment scheme for the closed-loop system towards its long-term equilibrium most. For, under such an adjustment scheme also unanticipated shocks to the system are dealt with best. Furthermore, this equilibrium seems to be a natural candidate that may be Pareto efficient (that is both players infer lower cost by playing this equilibrium). However, given the fact that we expect this to be a rare situation, we do not elaborate this subject here.

Finally note that the state variable s in the closed-loop system (16) does not depend directly on the value of i^E . This variable i^E has only an indirect influence on the closed-loop dynamics of the model, that is via the parameters in the cost functionals.

3.2 The Cooperative Case

The various interdependencies and spillovers between the two countries are not internalized if countries decide upon fiscal policies in a non-cooperative manner. In our case, national fiscal policies combined with initial disequilibria in intra-EU competitiveness imply important externalities. Domestic fiscal policies also impact on foreign output through the import channel. Any initial disequilibrium in intra-EU competitiveness, however, implies that both countries have opposite optimal policies. Therefore, national fiscal policy -while fostering domestic adjustment- at the same time increases the adjustment burden in the other economy. Coordination can help to reduce the working of such externalities caused by national fiscal policies in the presence of an initial disequilibrium in intra-EU competitiveness. Therefore, it is important to compare fiscal policies and macroeconomic outcomes under non-cooperative equilibria with outcomes under cooperation. The importance of surveillance and coordination of macroeconomic policies in the EU is stressed in the Maastricht Treaty which requires member states to regard their macroeconomic policies as a "matter of common concern" and to coordinate these within the Council of Ministers. In these ECOFIN meetings coordination and surveillance of macroeconomic policies has now been insti-

tutionalized.

Under cooperation fiscal policies are directed at minimizing a joined loss function, J^C , rather than at minimizing the individual national loss functions,

$$J^C = J^F + \omega J^{F*} \quad (17)$$

where ω is the Pareto constant that measures the relative weight attached to the losses of both players. One could assume that it is the outcome of an earlier bargaining problem that the two players have solved to determine the relative weights of the individual objectives in the cooperative design of fiscal policies. In that case the Nash-bargaining solution could be considered as the most natural outcome to such a bargaining problem associated with the cooperative decision making process.

We can rewrite the cooperative decision making problem in the standard format introduced earlier when analysing the Nash open-loop case as,

$$\begin{aligned} \min J^C &= \frac{1}{2} \int_0^\infty \{[x(t)^T \ u_1^T(t) \ u_2^T(t)]^T W \begin{bmatrix} x(t) \\ u_1(t) \\ u_2(t) \end{bmatrix}\} dt \\ \text{s.t. } \dot{x}(t) &= Ax(t) + B_1 u_1(t) + B_2 u_2(t), \\ x(0) &= x_0, \end{aligned} \quad (18)$$

where the positive definite matrix W is partitioned as,

$$W := \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}$$

in which Q, R and S represent 2x2 sub matrices that are given in Appendix II. Proceeding as before we use the Hamiltonian approach to calculate the optimal strategy (see Appendix II). After some lengthy calculations we find the following closed-loop system:

$$\dot{x}(t) = \begin{bmatrix} -\lambda & v \\ 0 & -\frac{1}{2}\theta \end{bmatrix} x(t), \quad (19)$$

where the adjustment speed λ is the positive square root that follows directly from (25) in Appendix II and v is a (in general non-zero) parameter that depends on the system parameters. Note that, different from the non-cooperative case, the variable i^E now has a direct impact on the closed-loop dynamics of the system.

Taking a closer look at λ as a function of the relative weight parameter ω , we see that it can be written as:

$$\lambda = \sqrt{\frac{\nu_1\omega + \nu_2(1+\omega)^2 - \nu_3(1+\omega^2)}{\nu_4\omega + \nu_5(1+\omega)^2}},$$

where ν_i are positive constants (see Appendix V, Table 6). Differentiation of this expression w.r.t. ω yields:

$$\lambda'(\omega) = \frac{1}{2\sqrt{\lambda}} \frac{(1-\omega^2)(\nu_1\nu_5 - \nu_2\nu_4 + \nu_3\nu_4 + 2\nu_5\nu_3)}{(\nu_4\omega + \nu_5(1+\omega)^2)^2}.$$

So, we conclude that, *ceteris paribus*, λ is maximized for $\omega = 1$ in case $\nu := \nu_1\nu_5 - \nu_2\nu_4 + \nu_3\nu_4 + 2\nu_5\nu_3$ is positive, and that λ is minimal for $\omega = 1$ in case $\nu < 0$. In Appendix III we show that $\nu < 0$ if and only if $(2ar(\phi_2 - \frac{1}{2}\eta) - b\phi_1)(a^2\mu(r^2 - 1) - \chi) - 2a^2b\phi_1\mu r(r+1) > 0$. In Figure 2, below, we illustrated the behavior of λ as a function of the coordination parameter ω .

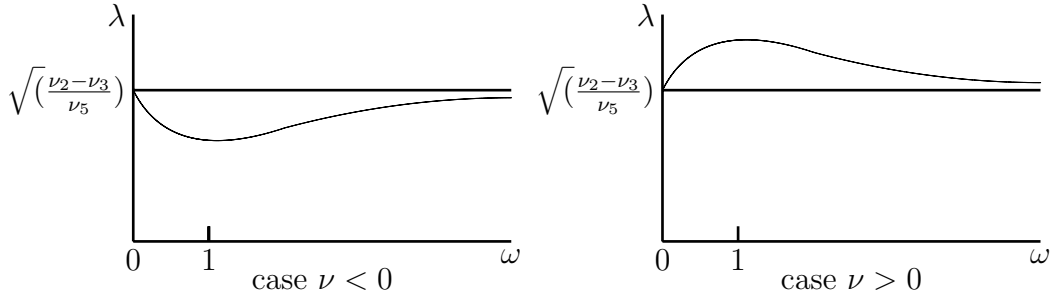


Figure 2

From this Figure we see that s converges as fast as possible to zero in the cooperative game if both cost-functionals have an equal weight in case $\nu > 0$. So, under these parameter conditions both players have an incentive to cooperate, since cooperation increases the adjustment speed of the closed-loop system (19) towards its long-term equilibrium. On the other hand in case $\nu < 0$, s converges as fast as possible to zero in case either $\omega = 0$ or $\omega = \infty$. One might expect that cooperation under these parameter conditions will be

much more difficult to achieve. For, whatever the value of the cooperation weight parameter ω is, both players observe that a different value of this parameter would increase the stability of their economy. Obviously, this is a desirable property as it implies that unanticipated shocks will have a less serious impact on the economy.

3.3 The Effect of Fiscal Stringency Conditions on λ

The impact of fiscal stringency is measured by the model parameter χ . In section 3.1 we analyzed already the consequences of fiscal stringency on the number of non-cooperative equilibria. We saw that if the model parameter r is smaller than one, fiscal stringency has no impact on the number of equilibria. There is always a unique equilibrium. However, in case $r > 1$ fiscal stringency does have an impact. If fiscal stringency conditions are either rather weak or very strong, again a unique equilibrium will occur, whereas if fiscal stringency is in between a lower and upper bound, χ_1 and χ_2 , either two or no equilibrium can occur.

In section 3.2 we showed that in case the sign of the parameter ν is negative, one may expect that cooperation will be difficult to achieve. In fact this happens if and only if $(2ar(\phi_2 - \frac{1}{2}\eta) - b\phi_1)(a^2\mu(r^2 - 1) - \chi) - 2a^2b\phi_1\mu r(r + 1) > 0$ or, stated differently in terms of the fiscal stringency measure χ , $\chi > \frac{a^2\mu(r+1)(2ar(r-1)+b\phi_1(r-\frac{1}{3}))}{b\phi_1-2ar(\phi_2-\frac{1}{2}\theta)}$.

In other words there is always a threshold after which, if fiscal stringency is increased even more, the realisation of a cooperative equilibrium will be more difficult to achieve. Tight fiscal stringency conditions imply that the domestic government is rather reluctant in using fiscal instruments to stabilize domestic output and prices. Since the foreign country has the same attitude both countries are very reluctant to help out the other country in the achievement of an optimal performance. Note that in case $r < \frac{1}{3}$, irrespective of the other parameter values, always $\nu < 0$ holds. In that case, foreign deficits have only a limited effect on domestic output (see (10)) and a cooperative equilibrium between both countries will be difficult to achieve if countries care about the internal stability of their economy (i.e. prefer a high adjustment speed λ).

Next, we analyze the impact of fiscal stringency conditions on the closed-loop dynamics of the system under both scenarios. In Table 3 we show the impact

of χ on the closed-loop dynamics of the model under the assumption that $r < 1$. Details of the calculations are given in Appendix IV.

Table 3

χ	Non-Cooperative	Cooperative
0	$\lambda = \frac{1}{2}\theta$	$\lambda = \frac{1}{2}\theta$
:	increasing	increasing/(decreasing)
large	$\lambda = -a_{uc}$	$\lambda = -a_{uc}$

Here $a_{uc} := \phi_2 - \frac{1}{2}\theta$. The table should be interpreted as follows. If χ increases, the corresponding λ for the non-cooperative case increases (monotonically) from $\frac{1}{2}\theta$ to $-a_{uc}$. For the cooperative case two different situations can occur depending on the sign of $\sigma := -b\phi_1(1+\omega)^2 + 2aa_{uc}(r\omega^2 - 2\omega + r)$. If $\sigma > 0$, λ will increase (monotonically) in the cooperative case too. In case $\sigma < 0$, λ will first increase towards its maximum value (larger than $-a_{uc}$) and then decrease to $-a_{uc}$. We illustrate both cases in Figure 3.

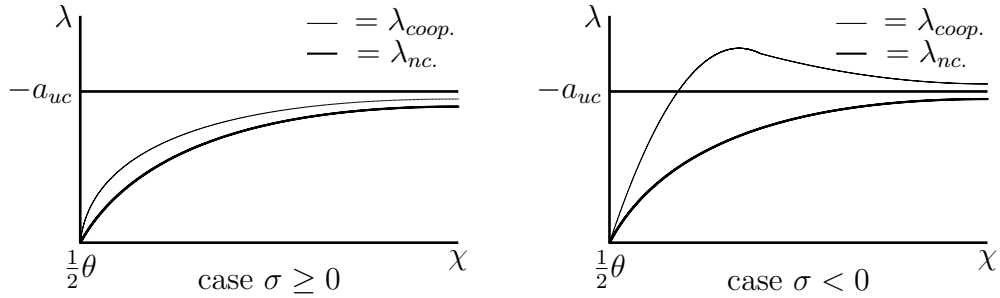


Figure 3

From Table 3 and Figure 3 we see that the adjustment speed of s towards its long-term equilibrium always increases in case fiscal deficits are taken more seriously, at least in the non-cooperative case. In the non-cooperative case the adjustment speed increases with higher values of χ : it reduces fiscal policy activism and therefore the negative externalities of national fiscal policies that occur when the dynamic system starts out of equilibrium. In the non-cooperative case these externalities are less activated by the players in case

the fiscal stringency requirements are imposed with more vigour, i.e. if χ is set higher, because with a higher valued χ the costs of high deficits and surpluses increase and policy activism is therefore reduced. E.g. if $s(0) > 0$, country 1 has an initial competitive advantage compared to country 2 and would like to reduce output and inflation by means of a fiscal surplus, this however also has a contractionary effect on country 2 whose economy is in recession already because of the initial disequilibrium and would suffer even more from a contractionary fiscal policy in country 1.

In the cooperative case the convergence speed is larger than in the non-cooperative case: in case of an initial disequilibrium of the state variable s , the negative spillovers from national fiscal policies are internalized when fiscal policies are set in a cooperative manner. In that case there exists, however, a threshold after which this convergence speed does not increase anymore (though it remains above that of the non-cooperative case). In case fiscal deficits are taken strongly into account, implying that χ is large, the impact on the convergence speed of s towards zero is almost the same in both scenario's. Note that this is also the case if in both scenario's fiscal deficits are (almost) neglected. Note also that if $\omega = 1$, σ is always positive and the monotonic relation between χ and λ applies. In case ω approaches either zero or infinity, implying that cooperative policy design is dominated by one country only, the case with $\sigma < 0$ applies and there exist some value for χ for which the adjustment speed is maximal.

Summarizing, we see that the adjustment speed of output price differential is higher in the cooperative case than in the non-cooperative one. Furthermore, if fiscal stringency would be a design parameter, we observe that for a high adjustment speed it is best to increase fiscal stringency conditions as much as possible in case countries are non-cooperative (that is try to prevent the individual players to intervene in the economy) and if countries are cooperative, there exists some intermediate level of fiscal stringency where the adjustment speed is maximal (provided the model parameters satisfy $\sigma < 0$). Moreover, in case fiscal deficits play either no role or a very important role it does not make any difference for the adjustment speed whether the countries cooperate or not.

3.4 Consequences of a European Federal Transfer System

It is well-known (see e.g. Weber, 1991, Bayoumi and Eichengreen, 1993, Bayoumi and Prasad, 1995, and Christodoulakis et al. 1995) that asymmetric macroeconomic shocks are fairly important in most countries of the European Union. Furthermore, Decressin et al. (1995), find that labor mobility is considerably smaller in the EU than in the US. Therefore, a European Fiscal Transfer System (EFTS) that aims at stabilizing asymmetric shocks in the EMU has been advocated by van der Ploeg (1991) and has been elaborated further by e.g. Italianer et al. (1993) and von Hagen (1995).

In this section we will include such an automatic stabilization rule into our model and analyze its consequences. In the context of EMU a fiscal transfer system operating through the budget of the EU seems to be the most realistic institutional framework, e.g., in the form of an EU-wide unemployment benefits scheme.

To that end we define net government expenditures as follows

$$\begin{aligned} g &:= f - z \\ g^* &:= f^* + z, \end{aligned}$$

where $z := \epsilon(y - y^*)$ is a net transfer from country 1 to country 2. The transfer system redirects demand from a country with a higher level of output to a country with a lower level of output. Thus, the transfer system contributes to automatic stabilization of intra EU divergences in output fluctuations.

Note that transfer systems in practice may induce negative incentives in that countries postpone adjustment measures in expectance of receiving transfers (consider e.g. the Mezzogiorno problem in the case of Italy where sustained transfers from North to South hampered structural adjustments in the South and created strong dependence from the South on the North). Our analysis -which deals with symmetric countries and cyclical fluctuations- disregards such incentive problems associated with fiscal transfer systems ¹.

The output equations (1), (2) then become

$$\begin{aligned} y(t) &= \delta s(t) - \gamma i_r(t) + \rho y^*(t) + \eta g(t) \\ y^*(t) &= -\delta s(t) - \gamma i_{r^*}(t) + \rho y(t) + \eta g^*(t) \end{aligned}$$

¹Welfare costs associated with a fiscal transfer system could be introduced by adding z to the welfare functions (13),(14) implying that higher transfers are more costly.

After some elementary calculations we have that this model can be rewritten into the previous framework, with the following redefinition of parameters:

$$a := \frac{\eta X}{X^2 - Z^2}; \quad b := \frac{\delta}{X + Z}; \quad k := \frac{\rho X}{Z}; \quad \phi_1 := \frac{\xi \eta}{X + Z}; \quad \phi_2 := \frac{-2\delta \xi}{X + Z},$$

in which $X := 1 - \gamma\xi + \eta\epsilon$ and $Z := \rho + \eta\epsilon$.

Using these parameter redefinitions all results obtained in the previous sections can be applied now. Some elementary calculations show that the parameters a, b, ϕ_1 and ϕ_2 are in absolute terms smaller and k is larger than in the original model, while c remains constant. The consequence of the introduction of an EFTS for our model are, see (10)-(12), that due to the direct output transfers divergences between both countries are automatically stabilized. In the EFTS case, therefore, less national fiscal policy activism is needed to stabilize output deviations. The role of the indirect stabilization mechanism via output price differentials (s) becomes less important. Consequently, initial output price differentials will be more persistent. In particular, if we recalculate the eigenvalue λ for the (non-)cooperative case we obtain the following result:

Table 4

χ	old model	model with EFTS
0	$\lambda = \frac{1}{2}\theta$	$\lambda = \frac{1}{2}\theta$
∞	$\lambda = \frac{2\delta\xi}{1-\gamma\xi+\rho} + \frac{1}{2}\theta$	$\lambda = \frac{2\delta\xi}{1-\gamma\xi+\rho+2\eta\epsilon} + \frac{1}{2}\theta$

From this table we see that the adjustment speed of output price differential towards its long-run equilibrium is lower in the EFTS model if fiscal stringency conditions are tight ($\chi \rightarrow \infty$).

4 Numerical Simulations with the Model

A numerical example is very useful to illustrate the main aspects of the workings of the model and the analytical results established in the preceding section. For the model parameters we take the following values

Table 5: Model Parameters

$\delta = 0.2$	$\zeta = 1$	$\theta = 0.15$
$\gamma = 0.4$	$\xi = 0.25$	$\omega = 1$
$\rho = 0.4$	$\alpha = 2$	$i^E(t) = \bar{i}^E = 0$
$\eta = 1$	$\beta = 5$	$s(0) = 0.05$
$\kappa = 1$	$\chi = 2, 5$	

Figure 4 graphs the adjustment dynamics that result in the non-cooperative (solid line) and cooperative case (dotted line).

[Insert Figure 4]

Under fiscal policy coordination, adjustment of the state variable, $s(t)$ (panel (e)), is faster than under non-cooperative fiscal policies. Compared with un-coordinated fiscal policies, fiscal policy coordination leads to less contractionary fiscal policies in country 1 (panel (a)) and to less expansionary fiscal policies in country 2 (panel (c)). A less contractionary fiscal policy in country 1 leads to more output fluctuations in country 1 (compared with the non-cooperative case) but contributes to stabilizing the economy of country 2 which is facing a recession. Under cooperation these externalities of fiscal policies are internalized in the design of optimal fiscal stabilization policies, producing more efficient policies than in the non-cooperative Nash case. This is also indicated by the welfare losses of both players, according to (13)-(14), which are calculated in the first row (I) of Table 6, both for the non-cooperative case (a) and cooperative case (b)

Table 6: Welfare losses

		J^F	J^{F*}	
I	(a)	0.412	0.412	non-coop
	(b)	0.368	0.368	coop
II	(a)	0.013	0.013	non-coop $\chi = 0$
	(b)	0.533	0.533	non-coop $\chi = 5$
III	(a)	0.011	0.011	coop $\chi = 0$
	(b)	0.474	0.474	coop $\chi = 5$
IV	(a)	0.368	0.368	coop $\omega = 1$
	(b)	0.302	0.470	coop $\omega = 0.5$
V	(a)	0.308	0.308	non-coop $\epsilon = 0.3$
	(b)	0.246	0.246	coop $\epsilon = 0.3$

- I: base scenario for (a) the non-cooperative and (b) the cooperative case.
 II: effect of either less (a) or stronger (b) interpretation of fiscal stringency in non-cooperative case.
 III: similar as in II but now for cooperative case.
 IV: effect of reducing bargaining weight player 2 in cooperative case.
 V: effect of fiscal transfer system ($\epsilon = 0.3$).

A stricter interpretation of the Maastricht restrictions on fiscal deficits reduces fiscal activism leading to more pronounced output fluctuations in the EMU. To analyze the effects of a higher degree of fiscal stringency on fiscal policies and macroeconomic adjustment in the EMU, we compare outcomes in two cases: (i) $\chi = 0$ (solid line) and (ii) $\chi = 5$ (dotted line). Figure 5 compares both cases under non-cooperative fiscal policy design. It turns out that the results for the cooperative case are similar. We, therefore, choose to plot these outcomes not separately.

[Insert Figure 5]

A higher degree of fiscal stringency reduces fiscal policy activism (panel (a) and (c)) in both countries both under non-cooperative and cooperative fiscal policy design, implying larger short-run output fluctuations (panel (b) and (d)), and consequently high welfare losses. As noted in section 3.3, the adjustment speed of the system dynamics increases when the degree of fiscal

stringency is increased. In our example, the effects from a change in fiscal stringency on fiscal deficits and output turned out to be somewhat stronger in the case of policy coordination (not shown). Rows II and III of Table 6 display the welfare losses that result in the non-cooperative and cooperative case with no fiscal stringency ($\chi = 0$, line (a)) and with high fiscal stringency ($\chi = 5$, line (b)).

In the case of fiscal policy coordination, the weighting parameter ω - that can also be interpreted as the relative bargaining strength of country 2 in the cooperative decision making process - plays an important role as it determines how much weight is attributed to the preferences of both countries in policy design. In Figure 6 the effect of reducing ω from 1 (solid lines) to 0.5 (dashed lines) is displayed. Note that we have assumed again that $\chi = 2.5$.

[Insert Figure 6]

With fiscal policies being more oriented to the needs of country 1 we in particular see a less expansionary fiscal policy in country 2 (panel (c)), which, therefore, faces larger output fluctuations (panel (d)), whereas country 1 features more stable output (panel (b)). The adjustment speed of the state variable $s(t)$ is slightly higher when ω is reduced to 0.5 (panel (e)). As noted in section 3.2, we are therefore in a case where $\nu < 0$. According to row IV of Table 6, welfare losses are redistributed from country 1 to country 2 when its bargaining power increases.

As discussed in section 3.4, a system of (federal) fiscal transfers may be a useful stabilization tool in a monetary union that features strong asynchronous business cycle fluctuations. To illustrate the workings of the transfer system we consider in Figure 7 the adjustment dynamics in case $\epsilon = 0.3$ (assuming again $\chi = 2.5$ and $\omega = 1$) in case of non-cooperative (solid lines) and cooperative (dashed lines) fiscal policies.

[Insert Figure 7]

Comparing with Figure 4, we find that the fiscal transfer system provides substantial automatic stabilization, resulting in lower output fluctuations (panel

(b) and (d)) and less need for fiscal stabilization at the national level (panel (a) and (c)). Not visible, but shown in section 3.4, is a reduced adjustment speed of the system dynamics when a federal transfer system is introduced. According to row V of Table 6, the transfer system enables to reduce welfare losses substantially compared to the base case of row I that features no fiscal transfer system. In that perspective, the introduction of the fiscal transfer system in a setting with asynchronous business cycle fluctuations and fiscal stringency conditions at the national level can be deemed as efficient.

5 Conclusions

This paper has analyzed the design of fiscal policies under EMU. Under EMU, countries loose monetary and exchange rate policies as macroeconomic stabilization tools. Therefore, the entire burden of stabilization is shifted to national fiscal policy adjustment. A symmetric two country model of the EMU with sluggish output and price adjustment in the short run was constructed. We modeled the design of fiscal stabilization policies in the EMU as a linear quadratic differential game between national fiscal authorities. In this game we analyzed the Nash open-loop and the cooperative equilibria. Of course, to keep our analysis tractable, we had to impose a number of restrictions. The next limitations of the analysis should be noted: (i) a symmetric two-country EMU was modeled and the interaction with non-EMU countries neglected; (ii) only Keynesian effects of fiscal policy were present in the model and intertemporal implications of the government budget constraint were ignored; (iii) we focussed on open-loop Nash equilibria and Pareto solutions assuming quadratic preferences and an infinite time planning horizon and (iv) a passive (non-strategic) ECB was assumed controlling the common nominal interest rate.

Within this framework it was shown how fiscal stabilization policies were directed at stabilization of the business cycle fluctuations. The effects of a set of externally imposed constraints on fiscal flexibility, such as those involved in the Stability Pact, were studied in detail. In general, the fiscal stringency criteria reduce the degree of fiscal policy activism and by that the degree of effective stabilization of output and prices in the EMU. In that perspective, these constraints are causing suboptimal macroeconomic policies. We also showed that fiscal stringency may have an impact on the number of non-cooperative equilibria and on the internal stability of the economy. For the

non-cooperative case stability increases if fiscal stringency increases, whereas for the cooperative case there may exist a threshold for fiscal stringency after which stability decreases again. For the cooperative case we looked in more detail at the effects of the bargaining power on the internal stability of the economy. It turned out that internal stability is either maximized or minimized with symmetric bargaining shares. Though, obviously, the sum of welfare losses is minimized if bargaining shares are symmetric the loss of internal stability might be a reason for the occurrence of a difficult bargaining process. We showed that such a situation always occurs if e.g. fiscal stringency exceeds a certain threshold or if foreign deficits have only a limited direct effect on domestic output.

Finally, the effects of a fiscal transfer system in the EMU were considered. We showed that such a system decreases the internal stability of the economy. On the other hand, we showed in an example that welfare costs may be considerably reduced using such a transfer system. So, when national fiscal policies are restricted such a transfer system may be a powerful stabilization instrument in the presence of business cycle divergences.

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Appendix

I. The noncooperative case

From our model the next values for the matrices follow:

$$A = \begin{bmatrix} \phi_2 - \frac{1}{2}\theta & 0 \\ 0 & -\frac{1}{2}\theta \end{bmatrix}; B_1 = \begin{bmatrix} -\phi_1 \\ 0 \end{bmatrix}; B_2 = \begin{bmatrix} \phi_1 \\ 0 \end{bmatrix}; Q_1 = \mu \begin{bmatrix} b^2 & -bc \\ -bc & c^2 \end{bmatrix};$$

$$P_1 = \mu \begin{bmatrix} ab \\ -ac \end{bmatrix}; L_1 = rP_1; R_{11} = \mu a^2 + \chi; N_1 = r\mu a^2; R_{21} = r^2\mu a^2$$

and

$$Q_2 = \mu \begin{bmatrix} b^2 & bc \\ bc & c^2 \end{bmatrix}; P_2 = r\mu \begin{bmatrix} -ab \\ -ac \end{bmatrix}; L_2 = \frac{1}{r}P_2; R_{12} = r^2\mu a^2; N_2 = r\mu a^2; R_{22} = \mu a^2 + \chi.$$

Assuming that the matrix

$$G := \begin{bmatrix} R_{11} & N_1 \\ N_2^T & R_{22} \end{bmatrix} = \begin{bmatrix} \mu a^2 + \chi & r\mu a^2 \\ r\mu a^2 & \mu a^2 + \chi \end{bmatrix}$$

is invertible we recall from Engwerda et al. (1999) the following result. Consider the case that we restrict ourselves to consider only control functions which yield finite cost and which, moreover, permit a feedback synthesis. Then, if both (A, B_1) and (A, B_2) are stabilizable and Q_i is positive definite w.r.t. the controllability subspace $\langle A, B_i \rangle$, the infinite-planning horizon two-player linear quadratic differential game has for every initial state an open-loop Nash equilibrium strategy if and only if there exist K_1 and K_2 that are solutions of the algebraic Riccati equations (see below) (ARE) satisfying the additional constraint that the eigenvalues of $A_{cl} := A - (B_1 B_2)G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix}$ are all situated in the left half complex plane. In that case, the strategy

$$\begin{pmatrix} u_1(t) \\ u_2(t) \end{pmatrix} = -G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix} \Phi(t, 0)x_0,$$

where $\Phi(t, 0)$ satisfies the transition equation $\dot{\Phi}(t, 0) = A_{cl}\Phi(t, 0)$; $\Phi(0, 0) = I$, is an open-loop Nash equilibrium strategy. Furthermore the corresponding cost are $x_0^T M_i x_0$, where M_i is the unique positive semi-definite solution of the Lyapunov equation

$$A_{cl}^T M_i + M_i A_{cl} + (I - G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix}) F_i \begin{pmatrix} I \\ (-G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix})^T \end{pmatrix} = 0.$$

The set of algebraic Riccati equations, (ARE), associated with this problem equals:

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix} = \left(\begin{pmatrix} -A^T & 0 \\ 0 & -A^T \end{pmatrix} + \begin{pmatrix} P_1 & L_1 \\ P_2 & L_2 \end{pmatrix} G^{-1} \begin{pmatrix} B_1^T & 0 \\ 0 & B_2^T \end{pmatrix} \right) \begin{pmatrix} K_1 \\ K_2 \end{pmatrix}$$

$$\begin{aligned}
& + \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} \left(-A + (B_1 B_2) G^{-1} \begin{pmatrix} P_1^T \\ L_2^T \end{pmatrix} \right) + \\
& \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} ((B_1 B_2) G^{-1} \begin{pmatrix} B_1^T & 0 \\ 0 & B_2^T \end{pmatrix}) \begin{pmatrix} K_1 \\ K_2 \end{pmatrix} + \\
& \left(\begin{pmatrix} P_1 & L_1 \\ P_2 & L_2 \end{pmatrix} G^{-1} \begin{pmatrix} P_1^T \\ L_2^T \end{pmatrix} - \begin{pmatrix} Q_1 \\ Q_2 \end{pmatrix} \right). \tag{20}
\end{aligned}$$

To calculate (both theoretically and numerically) the optimal policies in the open-loop Nash equilibrium we use the Hamiltonian approach.

In Engwerda et al. (1999) the next algorithm is provided to calculate all equilibria for this infinite planning horizon game.

Algorithm 1:

- Step 1: Calculate the Hamiltonian matrix $M :=$

$$\begin{pmatrix} -A + (B_1 B_2) G^{-1} \begin{pmatrix} P_1^T \\ L_2^T \end{pmatrix} & (B_1 B_2) G^{-1} \begin{pmatrix} B_1^T \\ 0 \end{pmatrix} & (B_1 B_2) G^{-1} \begin{pmatrix} 0 \\ B_2^T \end{pmatrix} \\ Q_1 - (P_1 L_1) G^{-1} \begin{pmatrix} P_1^T \\ L_2^T \end{pmatrix} & A^T - (P_1 L_1) G^{-1} \begin{pmatrix} B_1^T \\ 0 \end{pmatrix} & -(P_1 L_1) G^{-1} \begin{pmatrix} 0 \\ B_2^T \end{pmatrix} \\ Q_2 - (P_2 L_2) G^{-1} \begin{pmatrix} P_1^T \\ L_2^T \end{pmatrix} & -(P_2 L_2) G^{-1} \begin{pmatrix} B_1^T \\ 0 \end{pmatrix} & A^T - (P_2 L_2) G^{-1} \begin{pmatrix} 0 \\ B_2^T \end{pmatrix} \end{pmatrix}. \tag{21}$$

- Step 2: Calculate the spectrum of matrix M . If the number of positive eigenvalues (counted with algebraic multiplicities) is less than the number of state variables, n , goto Step 5.
- Step 3: Calculate all 2-dimensional M invariant subspaces \mathcal{K} for which $Re(\lambda) > 0$ for all $\lambda \in \sigma(M|_{\mathcal{K}})$. Calculate 3 2x2 matrices X , Y and Z such that $Im \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \mathcal{K}$. Consider only those \mathcal{K} for which X is invertible. If this set is empty, goto Step 5.
- Step 4: Let \mathcal{K} be an arbitrary element of the set determined in Step 3. Denote $K_1 := YX^{-1}$ and $K_2 := ZX^{-1}$. $\begin{pmatrix} u_1^*(t) \\ u_2^*(t) \end{pmatrix} = -G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix} \Phi(t, 0)x_0$

is an open-loop Nash equilibrium strategy. The spectrum of the corresponding closed-loop matrix $A - (B_1 \ B_2)G^{-1} \begin{pmatrix} P_1^T + B_1^T K_1 \\ L_2^T + B_2^T K_2 \end{pmatrix}$ equals $\sigma(-M|_{\mathcal{K}})$. If the set determined in step 3 contains more elements one can repeat this step to calculate different equilibria.

- Step 5: End of algorithm.

Denoting $(1-r)\mu a^2 + \chi$ by α_1 , $(1+r)\mu a^2 + \chi$ by α_2 and $\phi_2 - \frac{1}{2}\theta$ by a_{uc} elementary calculations yield

$$M = \begin{bmatrix} -a_{uc} - \frac{2\mu ab\phi_1}{\alpha_1} & 0 & \frac{\phi_1^2}{\alpha_1} & 0 & \frac{\phi_1^2}{\alpha_1} & 0 \\ 0 & \frac{1}{2}\theta & 0 & 0 & 0 & 0 \\ \frac{\mu b^2 \chi}{\alpha_1} & -\frac{\mu bc \chi}{\alpha_2} & a_{uc} + \frac{\mu ab\phi_1((1-r^2)a^2\mu + \chi)}{\alpha_1\alpha_2} & 0 & -\frac{r\mu ab\phi_1\chi}{\alpha_1\alpha_2} & 0 \\ -\frac{\mu bc \chi}{\alpha_1} & \frac{\mu c^2 \chi}{\alpha_2} & -\frac{\mu ac\phi_1((1-r^2)a^2\mu + \chi)}{\alpha_1\alpha_2} & -\frac{1}{2}\theta & \frac{r\mu ac\phi_1\chi}{\alpha_1\alpha_2} & 0 \\ \frac{\mu b^2 \chi}{\alpha_1} & \frac{\mu bc \chi}{\alpha_2} & -\frac{r\mu ab\phi_1\chi}{\alpha_1\alpha_2} & 0 & a_{uc} + \frac{\mu ab\phi_1((1-r^2)a^2\mu + \chi)}{\alpha_1\alpha_2} & 0 \\ \frac{\mu bc \chi}{\alpha_1} & \frac{\mu c^2 \chi}{\alpha_2} & -\frac{r\mu ac\phi_1\chi}{\alpha_1\alpha_2} & 0 & \frac{\mu ac\phi_1((1-r^2)a^2\mu + \chi)}{\alpha_1\alpha_2} & -\frac{1}{2}\theta \end{bmatrix};$$

The eigenvalues of this matrix are: $\{-\frac{1}{2}\theta, -\frac{1}{2}\theta, \frac{1}{2}\theta, p := -\frac{1}{2}\theta + \phi_2 + \frac{\mu ab\phi_1}{\alpha_1\alpha_2}((1+r)\chi + (1-r^2)a^2\mu), \lambda_1, \lambda_2\}$, where

$$\lambda_{1,2} = \frac{1}{2} \left\{ \frac{-(1+r)\mu ab\phi_1}{\alpha_1} \pm \sqrt{\left(\frac{-(1+r)\mu ab\phi_1}{\alpha_1} \right)^2 - \frac{4\alpha_3}{\alpha_1^2}}, \right\} \quad (22)$$

in which $\alpha_3 := -\{(a_{uc}\alpha_1^2 + 2\mu ab\phi_1\alpha_1)(a_{uc} + \frac{1}{\alpha_1}\mu ab\phi_1(1-r)) + 2\phi_1^2\mu b^2\chi\}$.

Note that the square root term always exists as a real number, since this term can be rewritten as the sum of two positive numbers:

$$\frac{1}{\alpha_1^2} (\{(-3+r)\mu ab\phi_1 - 2a_{uc}((1-r)\mu a^2 + \chi)\}^2 + 8\mu\chi b^2\phi_1^2)$$

It is easily verified that the first two entries of the eigenvector corresponding to the eigenvalue p are zero. Moreover, the first entry of the eigenvector corresponding to the eigenvalue $\frac{1}{2}\theta$ is always zero as is the second entry of the eigenvector corresponding to λ_i , $i = 1, 2$. From this immediately follows that the model has at most 2 different equilibria. Moreover, by calculating the exact structure of the eigenvalues corresponding to the eigenvalues $\frac{1}{2}\theta$ and λ_i , and using the above computational algorithm the closed-loop structure

can be determined, as summarized in (16).

Some elementary rewriting shows that α_3 can be rewritten as

$$-\frac{1}{4} \frac{\alpha_1}{(k+\rho)^2} \{ (4\delta\xi + \theta(k+\rho))^2 \chi + \frac{\mu\eta^2\theta(\theta k + 2\delta\xi)}{k-\rho} \}.$$

It is now easily verified that if $k > \rho$ the parameters a and α_1 are positive and α_3 is, consequently, negative. So, M has exactly 2 positive eigenvalues. In case $k < \rho$, then $a < 0$. So there will be exactly one equilibrium if either $\alpha_1 < 0$ and $(4\delta\xi + \theta(k+\rho))^2 \chi < \frac{\mu\eta^2\theta(\theta k + 2\delta\xi)}{\rho-k}$ or $\alpha_1 > 0$ and $(4\delta\xi + \theta(k+\rho))^2 \chi > \frac{\mu\eta^2\theta(\theta k + 2\delta\xi)}{\rho-k}$. Denoting $(4\delta\xi + \theta(k+\rho))^2 \chi$ by \bar{y}_1 and $\frac{\mu\eta^2\theta(\theta k + 2\delta\xi)}{\rho-k}$ by \bar{y}_2 , it is moreover easily verified that there exists no equilibrium in case $\alpha_1 < 0$ and $\bar{y}_1 > \bar{y}_2$, and that there are two equilibria in case $\alpha_1 > 0$ and $\bar{y}_1 < \bar{y}_2$. Using the definition of α_1 and denoting $\frac{\mu(\theta k + 2\delta\xi)\eta^2\theta}{(\rho-k)(4\delta\xi + \theta(k+\rho))^2}$ by χ_1 and $\frac{\rho-k}{k} \mu \left(\frac{\eta k}{k^2 - \rho^2} \right)^2$ by χ_2 we can rewrite these conditions in terms of inequalities that should be satisfied by the design parameter χ . That is, there is one equilibrium if either $\chi < \min(\chi_1, \chi_2)$ or $\chi > \max(\chi_1, \chi_2)$; there is no equilibrium if $\chi_1 < \chi < \chi_2$; and there are 2 equilibria if $\chi_2 < \chi < \chi_1$. We summarized these results in Table 2.

II. The cooperative case

From our model the next values for the matrices follow:

$$A = \begin{bmatrix} \phi_2 - \frac{1}{2}\theta & 0 \\ 0 & -\frac{1}{2}\theta \end{bmatrix}; \quad B := [B_1 \ B_2] = \begin{bmatrix} -\phi_1 & \phi_1 \\ 0 & 0 \end{bmatrix}; \quad \text{and}$$

$$W = \begin{bmatrix} (1+\omega)\mu b^2 & (\omega-1)\mu bc & (1-\omega r)\mu ab & (r-\omega)\mu ab \\ (-1+\omega)\mu bc & (1+\omega)\mu c^2 & (-1-\omega r)\mu ac & (-r-\omega)\mu ac \\ (1-\omega r)\mu ab & (-1-\omega r)\mu ac & (1+\omega r^2)\mu a^2 + \chi & r(1+\omega)\mu a^2 \\ (r-\omega)\mu ab & (-r-\omega)\mu ac & r(1+\omega)\mu a^2 & (r^2+\omega)\mu a^2 + \omega\chi \end{bmatrix}.$$

Factorization of W immediately yields then the following parameter values for the matrices Q , S and R :

$$Q = \begin{bmatrix} (1+\omega)\mu b^2 & (\omega-1)\mu bc \\ (-1+\omega)\mu bc & (1+\omega)\mu c^2 \end{bmatrix}; \quad S = \begin{bmatrix} (1-\omega r)\mu ab & (r-\omega)\mu ab \\ (-1-\omega r)\mu ac & (-r-\omega)\mu ac \end{bmatrix};$$

$$\text{and } R = \begin{bmatrix} (1 + \omega r^2)\mu a^2 + \chi & r(1 + \omega)\mu a^2 \\ r(1 + \omega)\mu a^2 & (r^2 + \omega)\mu a^2 + \omega\chi \end{bmatrix}.$$

Note that in our case matrix R is invertible. Furthermore, (A, B) is stabilizable and (Q, A) is detectable.

From Lancaster and Rodman (1995, chapter 16) we recall that the optimal policies that result, equal,

$$\begin{bmatrix} u_1^*(t) \\ u_2^*(t) \end{bmatrix} = -R^{-1} \begin{bmatrix} S^T + B^T K \end{bmatrix} x(t) \quad (23)$$

where K is the unique positive semi-definite solution of the algebraic Riccati equation

$$KBR^{-1}B^TK - K(A - BR^{-1}S^T) - (A - BR^{-1}S^T)^TK - (Q - SR^{-1}S^T) = 0. \quad (24)$$

The corresponding minimal cost are $x_0^TKx_0$.

To calculate the optimal policy for the cooperative game one can proceed now similarly as in algorithm 1 of Appendix I (see e.g. Lancaster and Rodman 1995, chapter 7). The only differences are that M must be replaced by

$$H := \begin{bmatrix} -(A - BR^{-1}S^T) & BR^{-1}B^T \\ Q - SR^{-1}S^T & (A - BR^{-1}S^T)^T \end{bmatrix};$$

step 3 yields a unique solution; and that $K := YX^{-1}$ is obtained similarly as in step 4 by considering the decomposition $Im \begin{pmatrix} X \\ Y \end{pmatrix} = \mathcal{K}$.

Substitution of the above mentioned parameter values into H yields after some tedious manipulations, the following eigenvalues for this Hamiltonian: $\{\frac{1}{2}\theta, -\frac{1}{2}\theta, \pm\lambda\}$, where:

$$\begin{aligned} \lambda^2 = & \frac{1}{4(\omega(\chi + \mu a^2(1 - r^2))^2 + r^2\mu a^2\chi(1 + \omega)^2)} \{ (a\mu)^2\omega\{2(1 - r^2)aa_{uc} \\ & + 4(1 + r)b\phi_1\}^2 + \mu\chi\{-2(1 + \omega^2)aa_{uc}(2(1 - r^2)aa_{uc} + 4(1 + r)b\phi_1) \\ & + (1 + \omega)^2(2aa_{uc} + 2b\phi_1)^2\} + 4\omega\chi^2a_{uc}^2 \}. \end{aligned} \quad (25)$$

By calculating the eigenvectors corresponding to the eigenvalues $\frac{1}{2}\theta$ and λ , and using algorithm 1 the closed-loop structure (19) results.

III. A detailed study of the parameter ν

By definition $\nu = \nu_1\nu_5 - \nu_2\nu_4 + \nu_3\nu_4 + 2\nu_5\nu_3$. This can be rewritten as

$$\begin{aligned}
\nu &= (\nu_1 + 2\nu_3)\nu_5 + (\nu_3 - \nu_2)\nu_4 \\
&= 4[2a\mu(1-r^2)aa_{uc} + 4a\mu(1+r)b\phi_1 + 2\chi a_{uc}]^2\mu\chi r^2 a^2 - \\
&\quad 16\mu\chi(b\phi_1 - raa_{uc})^2(\chi + \mu a^2(1-r^2))^2 \\
&= 16\mu\chi\{a^2r^2[a_{uc}(\chi + \mu a^2(1-r^2)) + 2a\mu b\phi_1(1+r)]^2 - \\
&\quad (b\phi_1 - raa_{uc})^2(\chi + \mu a^2(1-r^2))^2\} \\
&= -16\mu\chi b\phi_1(\chi + a^2\mu(1+r)^2)[(2ara_{uc} - b\phi_1)(a^2\mu(r^2 - 1) - \chi) - 2a^2b\phi_1\mu r(r+1)]
\end{aligned}$$

The last equality can be verified, e.g., by straightforward expansion of both sides of the equation and then comparing terms.

Since $16\mu\chi b\phi_1(\chi + a^2\mu(1+r)^2) > 0$, the conclusions concerning the sign of ν follow directly.

IV. Sensitivity analysis of the closed-loop eigenvalues w.r.t. χ

By substituting $\chi = 0$ and $\chi = \infty$ into the λ 's one obtains the numbers mentioned in Table 3.

To analyze the intermediate behavior we consider the derivative of both λ 's w.r.t. χ . First consider the non-cooperative case under the assumption that $r < 1$. Then the appropriate λ is $\lambda = \frac{1}{2\alpha_1}\{-c_1 + \sqrt{c_1^2 - 4\alpha_3}\}$, where $c_1 := (1+r)\mu ab\phi_1$ (see 22). For analysis purposes we rewrite α_1 as $q_1 + \chi$ and α_3 as $-\frac{1}{4}\alpha_1(p_1\chi + p_2)$ (with $q_1 := (1-r)\mu a^2$, $p_1 := \frac{4\delta\xi + \theta(k+\rho)^2}{(k+\rho)^2}$ and $p_2 := \frac{\mu\theta\eta^2(\theta k + 2\delta\xi)}{(k+\rho)^2(k-\rho)}$).

Next, we rewrite λ as

$$\begin{aligned}
\lambda &= \frac{1}{2\alpha_1} \frac{-4\alpha_3}{c_1 + \sqrt{c_1^2 - 4\alpha_3}} \\
&= \frac{1}{2} \frac{p_1\chi + p_2}{c_1 + \sqrt{c_1^2 - 4\alpha_3}}
\end{aligned}$$

So,

$$\begin{aligned}
2 \frac{d\lambda}{d\chi} &= \frac{p_1(c_1 + \sqrt{c_1^2 - 4\alpha_3}) - \frac{1}{2\sqrt{c_1^2 - 4\alpha_3}}(p_1\alpha_1 + p_1\chi + p_2)(p_1\chi + p_2)}{(c_1 + \sqrt{c_1^2 - 4\alpha_3})^2} \\
&= \frac{p_1c_1\sqrt{c_1^2 - 4\alpha_3} + p_1(c_1^2 - 4\alpha_3) - \frac{1}{2}(p_1\alpha_1 + p_1\chi + p_2)(p_1\chi + p_2)}{\sqrt{c_1^2 - 4\alpha_3}(c_1 + \sqrt{c_1^2 - 4\alpha_3})^2} \\
&= \frac{p_1c_1(\sqrt{c_1^2 - 4\alpha_3} + c_1) - \frac{1}{2}(p_1\chi + p_2)^2 + \frac{1}{2}p_1\alpha_1(p_1\chi + p_2)}{\sqrt{c_1^2 - 4\alpha_3}(c_1 + \sqrt{c_1^2 - 4\alpha_3})^2} \\
&= \frac{p_1c_1(\sqrt{c_1^2 - 4\alpha_3} + c_1) + \frac{1}{2}(p_1\chi + p_2)(p_1\alpha_1 - p_1\chi - p_2)}{\sqrt{c_1^2 - 4\alpha_3}(c_1 + \sqrt{c_1^2 - 4\alpha_3})^2} \\
&= \frac{p_1c_1(\sqrt{c_1^2 - 4\alpha_3} + c_1) + \frac{1}{2}(p_1\chi + p_2)(p_1q_1 - p_2)}{\sqrt{c_1^2 - 4\alpha_3}(c_1 + \sqrt{c_1^2 - 4\alpha_3})^2}
\end{aligned}$$

From this it is clear that $\frac{d\lambda}{d\chi} > 0$ if we can show that $p_1q_1 - p_2 > 0$. Substitution of the model parameters into this expression (see Table 6) yields (note that by assumption $r < 1$, i.e. $k > \rho$)

$$\begin{aligned}
\text{sgn}(p_1q_1 - p_2) &= \text{sgn}\left\{\frac{(4\delta\xi + \theta(k + \rho))^2}{(k + \rho)^2} \frac{k - \rho}{k} \mu \frac{(k\eta)^2}{(k^2 - \rho^2)^2} - \frac{\mu\theta\eta^2(\theta k + 2\delta\xi)}{(k + \rho)^2(k - \rho)}\right\} \\
&= \text{sgn}\left\{\frac{k}{(k + \rho)^2}(4\delta\xi + \theta(k + \rho))^2 - (\theta k + 2\delta\xi)\theta\right\} \\
&= \text{sgn}\left\{\frac{k}{(k + \rho)^2}(8\delta\xi\theta(k + \rho) + 16\delta^2\xi^2) - 2\delta\xi\theta\right\}
\end{aligned}$$

Next, we show that this last expression is always positive. To do so, we first note that since $k > \rho$, we have $2k > k + \rho$. Therefore, $\frac{k}{(k + \rho)}8\delta\xi\theta - 2\delta\xi\theta > \frac{1}{2}8\delta\xi\theta - 2\delta\xi\theta > 0$. Using this inequality, the claim is obvious now. Which proves the positiveness of $\frac{d\lambda}{d\chi}$ for the non-cooperative case. Next, we consider the cooperative case. Some elementary analysis shows that in that case the corresponding λ (see 25) can be rewritten as

$$\lambda = \sqrt{\frac{d_1\chi^2 + d_2\chi + d_3}{d_4\chi^2 + d_5\chi + d_6}},$$

where d_i , $i = 1, \dots, 6$ are pointed out in Table 6.

Differentiation w.r.t. χ yields:

$$\frac{d\lambda}{d\chi} = \frac{1}{2\sqrt{\lambda}} \frac{e_1\chi^2 + 2e_2\chi + e_3}{(d_4\chi^2 + d_5\chi + d_6)^2},$$

where e_i , $i = 1, 2, 3$ are simple expressions in d_i (see either Table 6 or below). To analyze this derivative we first consider the sign of the parameters e_2 and e_3 . By definition we have that

$$\begin{aligned} e_2 &= d_1d_6 - d_4d_3 \\ &= 64a^2\mu^2\omega^2b\phi_1(1+r)^2(-b\phi_1 + aa_{uc}(r-1)) \end{aligned}$$

Furthermore, by first substituting the appropriate model parameters into d_i and next comparing terms on both sides of the equality signs we obtain

$$\begin{aligned} e_3 &= d_2d_6 - d_3d_5 \\ &= 16\omega a^4b\phi_1\mu^3(r+1)^3[b\phi_1(1+2r\omega+\omega^2) - 3b\phi_1(r\omega^2+2\omega+r) + \\ &\quad 2aa_{uc}(r-1)(r\omega^2+r+2\omega)] \\ &= 16\omega a^4b\phi_1\mu^3(r+1)^3[b\phi_1(1-r)(1+\omega)^2 + 2(aa_{uc}(r-1) - b\phi_1)(r\omega^2+r+2\omega)]. \end{aligned}$$

From the above expressions we see that both e_2 and e_3 are positive if we can show that $(aa_{uc}(r-1) - b\phi_1) > 0$. Using the definition of these parameters it is easily verified that $(aa_{uc}(r-1) - b\phi_1) = \frac{\frac{1}{2}\theta\eta}{k+\rho}$, from which the above inequality follows. So, both $e_2 > 0$ and $e_3 > 0$.

Finally, we consider e_1 . Some elementary rewriting shows:

$$\begin{aligned} e_1 &= d_1d_5 - d_2d_4 \\ &= 16\omega\mu b\phi_1(-b\phi_1(1+\omega)^2 + 2aa_{uc}(r\omega^2 - 2\omega + r)) \end{aligned}$$

So, denoting $-b\phi_1(1+\omega)^2 + 2aa_{uc}(r\omega^2 - 2\omega + r)$ by σ , we have that $e_1 = 16\omega\mu b\phi_1\sigma$.

Note that the sign of the derivative is completely determined by the sign of $e_1\chi^2 + 2e_2\chi + e_3$. Using the above derived information concerning the signs of e_i , $i = 1, 2, 3$ it is clear that if $\sigma > 0$, $\frac{d\lambda}{d\chi} > 0$ for all $\chi > 0$, and that if $\sigma < 0$, $\frac{d\lambda}{d\chi}$ will be positive for small χ and becomes negative if χ is large. From this the conclusions w.r.t. the behavior of λ as a function of χ summarized in Table 3 and Figure 3, respectively, are obvious then.

V. List of parameters

Table 6

name	value
a	$\frac{k\eta}{k^2-\rho^2}$
a_{uc}	$\phi_2 - \frac{1}{2}\theta$
α_1	$(1-r)\mu a^2 + \chi$
α_2	$(1+r)\mu a^2 + \chi$
α_3	$-((a_{uc}\alpha_1^2 + 2\mu ab\phi_1\alpha_1)(a_{uc} + \frac{1}{\alpha_1}\mu ab\phi_1(1-r)) + 2\phi_1^2\mu b^2\chi)$
b	$\frac{\delta}{k+\rho}$
c	$\frac{\gamma}{k-\rho}$
d_1	$4\omega a_{uc}^2$
d_2	$\mu\{-2(1+\omega^2)aa_{uc}(2(1-r^2)aa_{uc} + 4(1+r)b\phi_1) + (1+\omega)^2(2aa_{uc} + 2b\phi_1)^2\}$
d_3	$(a\mu)^2\omega\{2(1-r^2)aa_{uc} + 4(1+r)b\phi_1\}^2$
d_4	4ω
d_5	$8\omega\mu a^2(1-r^2) + 4r^2\mu a^2(1+\omega)^2$
d_6	$4\omega\mu^2 a^4(1-r^2)^2$
e_1	$d_1d_5 - d_2d_4$
e_2	$d_1d_6 - d_4d_3$
e_3	$d_2d_6 - d_3d_5$
k	$1 - \gamma\xi$
μ	$\alpha\xi^2 + \beta$
ν_1	$(a\mu)^2\{2(1-r^2)aa_{uc} + 4(1+r)b\phi_1\}^2 + 4\chi^2 a_{uc}^2$
ν_2	$\mu\chi(2aa_{uc} + 2b\phi_1)^2$
ν_3	$2\mu\chi aa_{uc}(2(1-r^2)aa_{uc} + 4(1+r)b\phi_1)$
ν_4	$4(\chi + \mu a^2(1-r^2))^2$
ν_5	$4\mu\chi r^2 a^2$
ν	$\nu_1\nu_5 - \nu_2\nu_4 + \nu_3\nu_4 + 2\nu_5\nu_3$
p_1	$\frac{(4\delta\xi + \theta(k+\rho))^2}{(k+\rho)^2}$
p_2	$\frac{\mu\theta\eta^2(\theta k + 2\delta\xi)}{(k+\rho)^2(k-\rho)}$
ϕ_1	$\frac{\xi\eta}{k+\rho}$
ϕ_2	$\frac{-2\delta\xi}{k+\rho}$
q_1	$(1-r)\mu a^2$
r	$\frac{\rho}{k}$
σ	$-b\phi_1(1+\omega)^2 + 2aa_{uc}(r\omega^2 - 2\omega + r)$

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